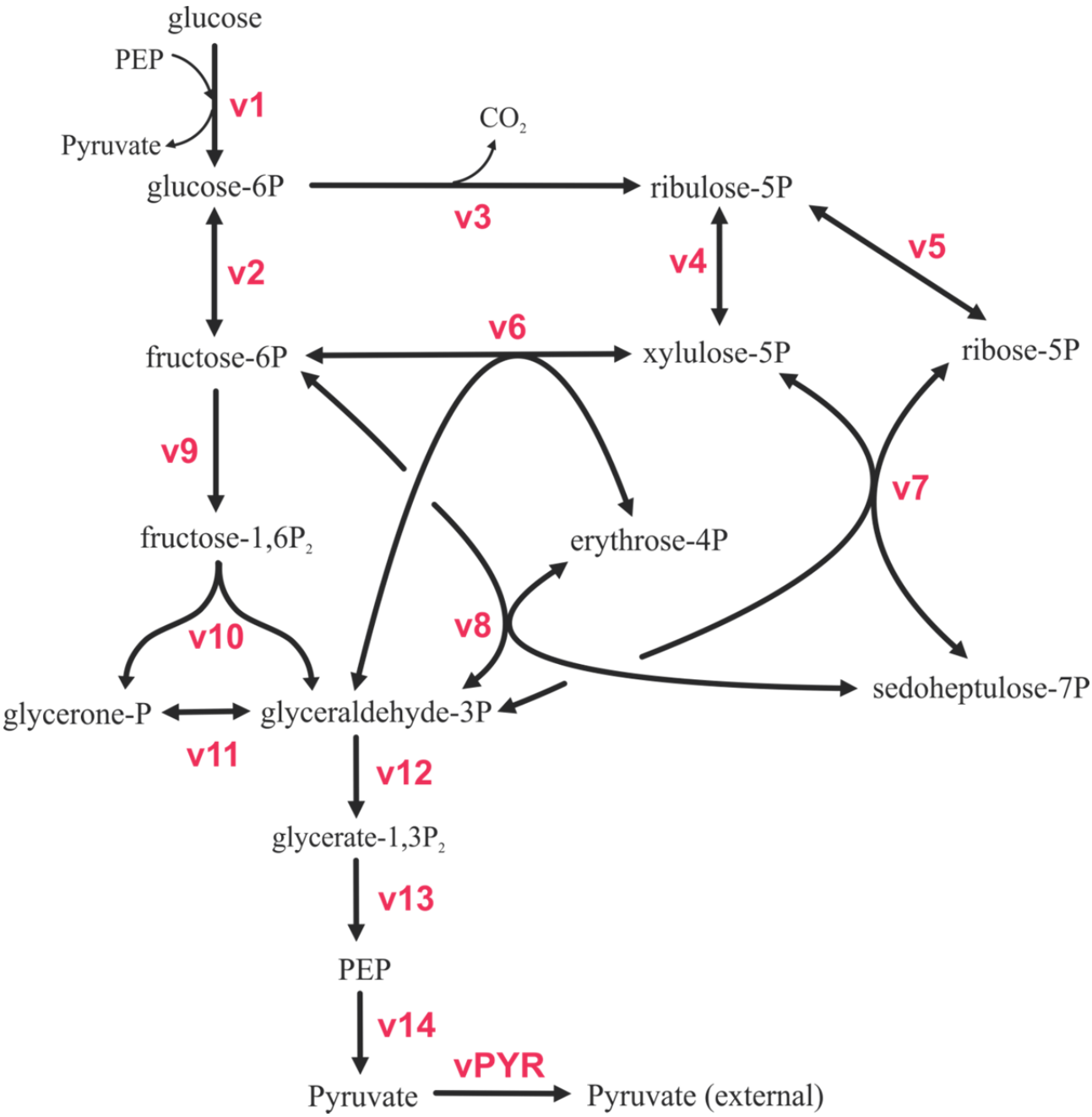
Problem E4

1) Draw Flux Map



2) Overall fluxes of metabolites

we know v1, v2, v3, so we would like to write all other fluxes in terms of v1, v2, v3

a) 0 = v1 – v2 – v3
b) 0 = v3 – v4 – v5
c) 0 = v4 – v6 – v7
d) 0 = v5 – v7
e) 0 = v7 – v8
f) 0 = v8 – v6
g) 0 = v2 + v6 + v8 – v9
h) 0 = v9 – v10
i) 0 = v10 – v11
j) 0 = v10 + v11 + v6 + v7 – v8 – v12
k) 0 = v12 – v13
l) 0 = v13 – v14 + v1
m) vpyr = v14 + v1

from d), e), f): v5 = v6 = v7 = v8 → use v8 everywhere
from h), i): v9 = v10 = v11 → use v11 everywhere
from k): v12 = v13 → use v13 everywhere

b) 0 = v3 – v4 – v8
c) 0 = v4 – v8 – v8 → v4 = 2v8
g) 0 = v2 + v8 + v8 – v11
j) 0 = v11 + v11 + v8 + v8 – v8 – v13
l) 0 = v13 – v14 – v1
m) vpyr = v14 + v1

b) 0 = v3 – 2v8 – v8 → <u>v8 = (1/3)v3</u> and <u>v4 = (2/3)v3</u>
g) v11 = v2 + 2v8 → <u>v11 = v2 + (2/3)v3</u>
j) v13 = 2v11 + v8 → v13 = 2[v2+(2/3)v3] +(1/3)v3 → <u>v13 = 2v2 + (5/3)v3</u>
l) v14 = v13 – v1 → <u>v14 = 2v2 + (5/3)v3 – v1</u>
m) <u>vpyr = 2v2 + (5/3)v3</u>

In summary:

v4 = (2/3)v3
v5 = (1/3)v3
v6 = (1/3)v3
v7 = (1/3)v3
v8 = (1/3)v3
v9 = v2 + (2/3)v3
v10 = v2 + (2/3)v3
v11 = v2 + (2/3)v3
v12 = 2v2 + (5/3)v3
v13 = 2v2 + (5/3)v3
v14 = 2v2 + (5/3)v3 – v1
vpyr = 2v2 + (5/3)v3

3) Atom Balances

Write a balance around each node, considering the individual atoms participating in the conversion at that node. In this particular problem, we are not considering the reversibility of fluxes at v2, v6, v7, v8.

G6P = glucose-6P

$$0 = \begin{pmatrix} \mathbf{Glu(1)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix} \boldsymbol{v1} - \begin{pmatrix} \mathbf{G6P(1)} \\ \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} \boldsymbol{v2} - \begin{pmatrix} \mathbf{G6P(1)} \\ \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} \boldsymbol{v3}$$

Ru5P = ribulose-5P

$$0 = \begin{pmatrix} \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} \boldsymbol{v3} - \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \boldsymbol{v4} - \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \boldsymbol{v5}$$

X5P = xylulose-5P

$$0 = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \boldsymbol{v4} - \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{X5P(3)} \\ \mathbf{X5P(4)} \\ \mathbf{X5P(5)} \end{pmatrix} \boldsymbol{v6} - \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{X5P(3)} \\ \mathbf{X5P(4)} \\ \mathbf{X5P(5)} \end{pmatrix} \boldsymbol{v7}$$

R5P = ribose-5P

$$0 = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \boldsymbol{v5} - \begin{pmatrix} \mathbf{R5P(1)} \\ \mathbf{R5P(2)} \\ \mathbf{R5P(3)} \\ \mathbf{R5P(4)} \\ \mathbf{R5P(5)} \end{pmatrix} \boldsymbol{v7}$$

S7P = sedoheptulose-7P

$$0 = \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{R5P(1)} \\ \mathbf{R5P(2)} \\ \mathbf{R5P(3)} \\ \mathbf{R5P(4)} \\ \mathbf{R5P(5)} \end{pmatrix} \boldsymbol{v7} - \begin{pmatrix} \mathbf{S7P(1)} \\ \mathbf{S7P(2)} \\ \mathbf{S7P(3)} \\ \mathbf{S7P(4)} \\ \mathbf{S7P(5)} \\ \mathbf{S7P(6)} \\ \mathbf{S7P(7)} \end{pmatrix} \boldsymbol{v8}$$

E4P = erythrose-4P

$$0 = \begin{pmatrix} \mathbf{S7P(4)} \\ \mathbf{S7P(5)} \\ \mathbf{S7P(6)} \\ \mathbf{S7P(7)} \end{pmatrix} \boldsymbol{v8} - \begin{pmatrix} \mathbf{E4P(1)} \\ \mathbf{E4P(2)} \\ \mathbf{E4P(3)} \\ \mathbf{E4P(4)} \end{pmatrix} \boldsymbol{v6}$$

F6P = fructose-6P

$$0 = \begin{pmatrix} \mathbf{G6P(1)} \\ \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} \boldsymbol{v2} + \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{E4P(1)} \\ \mathbf{E4P(2)} \\ \mathbf{E4P(3)} \\ \mathbf{E4P(4)} \end{pmatrix} \boldsymbol{v6} + \begin{pmatrix} \mathbf{S7P(1)} \\ \mathbf{S7P(2)} \\ \mathbf{S7P(3)} \\ \mathbf{Gly3P(1)} \\ \mathbf{Gly3P(2)} \\ \mathbf{Gly3P(3)} \end{pmatrix} \boldsymbol{v8} - \begin{pmatrix} \mathbf{F6P(1)} \\ \mathbf{F6P(2)} \\ \mathbf{F6P(3)} \\ \mathbf{F6P(4)} \\ \mathbf{F6P(5)} \\ \mathbf{F6P(6)} \end{pmatrix} \boldsymbol{v9}$$

FDP = fructose-1,6P_2

$$0 = \begin{pmatrix} \mathbf{F6P(1)} \\ \mathbf{F6P(2)} \\ \mathbf{F6P(3)} \\ \mathbf{F6P(4)} \\ \mathbf{F6P(5)} \\ \mathbf{F6P(6)} \end{pmatrix} \boldsymbol{v9} - \begin{pmatrix} \mathbf{FDP(1)} \\ \mathbf{FDP(2)} \\ \mathbf{FDP(3)} \\ \mathbf{FDP(4)} \\ \mathbf{FDP(5)} \\ \mathbf{FDP(6)} \end{pmatrix} \boldsymbol{v10}$$

GlP = glycerone-P

$$0 = \begin{pmatrix} \mathbf{FDP(1)} \\ \mathbf{FDP(2)} \\ \mathbf{FDP(3)} \end{pmatrix} \boldsymbol{v10} - \begin{pmatrix} \mathbf{GlP(1)} \\ \mathbf{GlP(2)} \\ \mathbf{GlP(3)} \end{pmatrix} \boldsymbol{v11}$$

Gly3P = glyceraldehyde-3P

$$0 = \begin{pmatrix} \mathbf{FDP(4)} \\ \mathbf{FDP(5)} \\ \mathbf{FDP(6)} \end{pmatrix} \boldsymbol{v10} + \begin{pmatrix} \mathbf{GlP(3)} \\ \mathbf{GlP(2)} \\ \mathbf{GlP(1)} \end{pmatrix} \boldsymbol{v11} + \begin{pmatrix} \mathbf{X5P(3)} \\ \mathbf{X5P(4)} \\ \mathbf{X5P(5)} \end{pmatrix} \boldsymbol{v6} + \begin{pmatrix} \mathbf{X5P(3)} \\ \mathbf{X5P(4)} \\ \mathbf{X5P(5)} \end{pmatrix} \boldsymbol{v7} - \begin{pmatrix} \mathbf{Gly3P(1)} \\ \mathbf{Gly3P(2)} \\ \mathbf{Gly3P(3)} \end{pmatrix} \boldsymbol{v8} - \begin{pmatrix} \mathbf{Gly3P(1)} \\ \mathbf{Gly3P(2)} \\ \mathbf{Gly3P(3)} \end{pmatrix} \boldsymbol{v12}$$

GDP = glycerate-1,3P_2

$$0 = \begin{pmatrix} \mathbf{Gly3P(1)} \\ \mathbf{Gly3P(2)} \\ \mathbf{Gly3P(3)} \end{pmatrix} \boldsymbol{v12} - \begin{pmatrix} \mathbf{GDP(1)} \\ \mathbf{GDP(2)} \\ \mathbf{GDP(3)} \end{pmatrix} \boldsymbol{v13}$$

PEP = phosphoenol pyruvate

$$0 = \begin{pmatrix} \mathbf{GDP(1)} \\ \mathbf{GDP(2)} \\ \mathbf{GDP(3)} \end{pmatrix} \boldsymbol{v13} - \begin{pmatrix} \mathbf{PEP(1)} \\ \mathbf{PEP(2)} \\ \mathbf{PEP(3)} \end{pmatrix} \boldsymbol{v1} - \begin{pmatrix} \mathbf{PEP(1)} \\ \mathbf{PEP(2)} \\ \mathbf{PEP(3)} \end{pmatrix} \boldsymbol{v14}$$

PYR = pyruvate

$$0 = \begin{pmatrix} \mathbf{PEP(1)} \\ \mathbf{PEP(2)} \\ \mathbf{PEP(3)} \end{pmatrix} \boldsymbol{v1} + \begin{pmatrix} \mathbf{PEP(1)} \\ \mathbf{PEP(2)} \\ \mathbf{PEP(3)} \end{pmatrix} \boldsymbol{v14} - \begin{pmatrix} \mathbf{PYR(1)} \\ \mathbf{PYR(2)} \\ \mathbf{PYR(3)} \end{pmatrix} \boldsymbol{v\mathbf{PYR}}$$

Note that these equations can be solved within a computer program. They do not need to be simplified.

4) Simplify. In this fairly simple case, the enrichment for each one of the metabolites can be related to the glucose enrichment.

G6P

$$\begin{pmatrix} \mathbf{G6P(1)} \\ \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(1)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix} \frac{\boldsymbol{v1}}{(\boldsymbol{v2}+\boldsymbol{v3})} = \begin{pmatrix} \mathbf{Glu(1)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

Ru5P

$$\begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} \frac{\boldsymbol{v3}}{(\boldsymbol{v4}+\boldsymbol{v5})} = \begin{pmatrix} \mathbf{G6P(2)} \\ \mathbf{G6P(3)} \\ \mathbf{G6P(4)} \\ \mathbf{G6P(5)} \\ \mathbf{G6P(6)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

X5P

$$\begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{X5P(3)} \\ \mathbf{X5P(4)} \\ \mathbf{X5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \frac{\boldsymbol{v3}}{(\boldsymbol{v4}+\boldsymbol{v5})} = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

R5P

$$\begin{pmatrix} \mathbf{R5P(1)} \\ \mathbf{R5P(2)} \\ \mathbf{R5P(3)} \\ \mathbf{R5P(4)} \\ \mathbf{R5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} \frac{\boldsymbol{v5}}{\boldsymbol{v7}} = \begin{pmatrix} \mathbf{Ru5P(1)} \\ \mathbf{Ru5P(2)} \\ \mathbf{Ru5P(3)} \\ \mathbf{Ru5P(4)} \\ \mathbf{Ru5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

S7P

$$\begin{pmatrix} \mathbf{S7P(1)} \\ \mathbf{S7P(2)} \\ \mathbf{S7P(3)} \\ \mathbf{S7P(4)} \\ \mathbf{S7P(5)} \\ \mathbf{S7P(6)} \\ \mathbf{S7P(7)} \end{pmatrix} = \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{R5P(1)} \\ \mathbf{R5P(2)} \\ \mathbf{R5P(3)} \\ \mathbf{R5P(4)} \\ \mathbf{R5P(5)} \end{pmatrix} \frac{\boldsymbol{v7}}{\boldsymbol{v8}} = \begin{pmatrix} \mathbf{X5P(1)} \\ \mathbf{X5P(2)} \\ \mathbf{R5P(1)} \\ \mathbf{R5P(2)} \\ \mathbf{R5P(3)} \\ \mathbf{R5P(4)} \\ \mathbf{R5P(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

E4P

$$\begin{pmatrix} \mathbf{E4P(1)} \\ \mathbf{E4P(2)} \\ \mathbf{E4P(3)} \\ \mathbf{E4P(4)} \end{pmatrix} = \begin{pmatrix} \mathbf{S7P(4)} \\ \mathbf{S7P(5)} \\ \mathbf{S7P(6)} \\ \mathbf{S7P(7)} \end{pmatrix} \frac{\boldsymbol{v8}}{\boldsymbol{v6}} = \begin{pmatrix} \mathbf{S7P(4)} \\ \mathbf{S7P(5)} \\ \mathbf{S7P(6)} \\ \mathbf{S7P(7)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix}$$

F6P

$$\begin{pmatrix} F6P(1) \\ F6P(2) \\ F6P(3) \\ F6P(4) \\ F6P(5) \\ F6P(6) \end{pmatrix} = \begin{pmatrix} G6P(1) \\ G6P(2) \\ G6P(3) \\ G6P(4) \\ G6P(5) \\ G6P(6) \end{pmatrix} \frac{v2}{v9} + \begin{pmatrix} X5P(1) \\ X5P(2) \\ E4P(1) \\ E4P(2) \\ E4P(3) \\ E4P(4) \end{pmatrix} \frac{v6}{v9} + \begin{pmatrix} S7P(1) \\ S7P(2) \\ S7P(3) \\ Gly3P(1) \\ Gly3P(2) \\ Gly3P(3) \end{pmatrix} \frac{v8}{v9}$$

$$\begin{pmatrix} F6P(1) \\ F6P(2) \\ F6P(3) \\ F6P(4) \\ F6P(5) \\ F6P(6) \end{pmatrix} = \begin{pmatrix} Glu(1) \\ Glu(2) \\ Glu(3) \\ Glu(4) \\ Glu(5) \\ Glu(6) \end{pmatrix} \frac{v2}{v9} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \\ Glu(3) \\ Glu(4) \\ Glu(5) \end{pmatrix} \frac{v6}{v9} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \\ Gly3P(1) \\ Gly3P(2) \\ Gly3P(3) \end{pmatrix} \frac{v8}{v9}$$

FDP

$$\begin{pmatrix} FDP(1) \\ FDP(2) \\ FDP(3) \\ FDP(4) \\ FDP(5) \\ FDP(6) \end{pmatrix} = \begin{pmatrix} F6P(1) \\ F6P(2) \\ F6P(3) \\ F6P(4) \\ F6P(5) \\ F6P(6) \end{pmatrix} \frac{v9}{v10} = \begin{pmatrix} Glu(1) \\ Glu(2) \\ Glu(3) \\ Glu(4) \\ Glu(5) \\ Glu(6) \end{pmatrix} \frac{v2}{v10} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \\ Glu(3) \\ Glu(4) \\ Glu(5) \end{pmatrix} \frac{v6}{v10} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \\ Gly3P(1) \\ Gly3P(2) \\ Gly3P(3) \end{pmatrix} \frac{v8}{v10}$$

GlP

$$\begin{pmatrix} GlP(1) \\ GlP(2) \\ GlP(3) \end{pmatrix} = \begin{pmatrix} FDP(1) \\ FDP(2) \\ FDP(3) \end{pmatrix} \frac{v10}{v11} = \begin{pmatrix} Glu(1) \\ Glu(2) \\ Glu(3) \end{pmatrix} \frac{v2}{v11} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \end{pmatrix} \frac{v6}{v11} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \end{pmatrix} \frac{v8}{v11}$$

Gly3P

$$\begin{pmatrix} Gly3P(1) \\ Gly3P(2) \\ Gly3P(3) \end{pmatrix} = \begin{pmatrix} FDP(4) \\ FDP(5) \\ FDP(6) \end{pmatrix} \frac{v10}{v8+v12} + \begin{pmatrix} GlP(3) \\ GlP(2) \\ GlP(1) \end{pmatrix} \frac{v11}{v8+v12} + \begin{pmatrix} X5P(3) \\ X5P(4) \\ X5P(5) \end{pmatrix} \frac{v6+v7}{v8+v12}$$

$$\begin{pmatrix} Gly3P(1) \\ Gly3P(2) \\ Gly3P(3) \end{pmatrix} = \begin{pmatrix} FDP(4) \\ FDP(5) \\ FDP(6) \end{pmatrix} \frac{v10}{v8+v12} + \begin{pmatrix} Glu(3) \\ Glu(2) \\ Glu(1) \end{pmatrix} \frac{v2}{v8+v12} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \end{pmatrix} \frac{v6}{v8+v12} + \begin{pmatrix} Glu(2) \\ Glu(3) \\ Glu(2) \end{pmatrix} \frac{v8}{v8+v12} + \begin{pmatrix} Glu(4) \\ Glu(5) \\ Glu(6) \end{pmatrix} \frac{v6+v7}{v8+v12}$$

Insert expression for FDP into equation for Gly3P:

$$\begin{pmatrix}\mathbf{Gly3P(1)}\\ \mathbf{Gly3P(2)}\\ \mathbf{Gly3P(3)}\end{pmatrix} = \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v2}{v8+v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(4)}\\ \mathbf{Glu(5)}\end{pmatrix}\frac{v6}{v8+v12} + \begin{pmatrix}\mathbf{Gly3P(1)}\\ \mathbf{Gly3P(2)}\\ \mathbf{Gly3P(3)}\end{pmatrix}\frac{v8}{v8+v12}$$
$$+ \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(2)}\\ \mathbf{Glu(1)}\end{pmatrix}\frac{v2}{v8+v12} + \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v6}{v8+v12} + \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v8}{v8+v12}$$
$$+ \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v6+v7}{v8+v12}$$

$$\begin{pmatrix}\mathbf{Gly3P(1)}\\ \mathbf{Gly3P(2)}\\ \mathbf{Gly3P(3)}\end{pmatrix}\left(1-\frac{v8}{v8+v12}\right)$$
$$= \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v2}{v8+v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(4)}\\ \mathbf{Glu(5)}\end{pmatrix}\frac{v6}{v8+v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(2)}\\ \mathbf{Glu(1)}\end{pmatrix}\frac{v2}{v8+v12}$$
$$+ \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v6}{v8+v12} + \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v8}{v8+v12} + \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v6+v7}{v8+v12}$$

$$\begin{pmatrix}\mathbf{Gly3P(1)}\\ \mathbf{Gly3P(2)}\\ \mathbf{Gly3P(3)}\end{pmatrix} = \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v2}{v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(4)}\\ \mathbf{Glu(5)}\end{pmatrix}\frac{v6}{v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(2)}\\ \mathbf{Glu(1)}\end{pmatrix}\frac{v2}{v12} + \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v6}{v12}$$
$$+ \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v8}{v12} + \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v6+v7}{v12}$$

$$\begin{pmatrix}\mathbf{Gly3P(1)}\\ \mathbf{Gly3P(2)}\\ \mathbf{Gly3P(3)}\end{pmatrix} = \begin{pmatrix}\mathbf{Glu(4)}\\ \mathbf{Glu(5)}\\ \mathbf{Glu(6)}\end{pmatrix}\frac{v2+v6+v7}{v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(4)}\\ \mathbf{Glu(5)}\end{pmatrix}\frac{v6}{v12} + \begin{pmatrix}\mathbf{Glu(3)}\\ \mathbf{Glu(2)}\\ \mathbf{Glu(1)}\end{pmatrix}\frac{v2}{v12}$$
$$+ \begin{pmatrix}\mathbf{Glu(2)}\\ \mathbf{Glu(3)}\\ \mathbf{Glu(2)}\end{pmatrix}\frac{v6+v8}{v12}$$

GDP/PEP/PYR

$$\begin{pmatrix} \mathbf{PYR(1)} \\ \mathbf{PYR(2)} \\ \mathbf{PYR(3)} \end{pmatrix} = \begin{pmatrix} \mathbf{Gly3P(1)} \\ \mathbf{Gly3P(2)} \\ \mathbf{Gly3P(3)} \end{pmatrix} \frac{\boldsymbol{v12}}{\boldsymbol{vPYR}}$$

$$\begin{pmatrix} \mathbf{PYR(1)} \\ \mathbf{PYR(2)} \\ \mathbf{PYR(3)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix} \frac{\boldsymbol{v2 + v6 + v7}}{\boldsymbol{vPYR}} + \begin{pmatrix} \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \end{pmatrix} \frac{\boldsymbol{v6}}{\boldsymbol{vPYR}} + \begin{pmatrix} \mathbf{Glu(3)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(1)} \end{pmatrix} \frac{\boldsymbol{v2}}{\boldsymbol{vPYR}} + \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(2)} \end{pmatrix} \frac{\boldsymbol{v6 + v8}}{\boldsymbol{vPYR}}$$

Note that because of the definitions of the fluxes, in terms of v2 and v3:

$$\begin{pmatrix} \mathbf{PYR(1)} \\ \mathbf{PYR(2)} \\ \mathbf{PYR(3)} \end{pmatrix} = \begin{pmatrix} \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \\ \mathbf{Glu(6)} \end{pmatrix} \left(\frac{\boldsymbol{v2} + \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{v3}}{\mathbf{2}\boldsymbol{v2} + \frac{\mathbf{5}}{\mathbf{3}}\boldsymbol{v3}} \right) + \begin{pmatrix} \mathbf{Glu(3)} \\ \mathbf{Glu(4)} \\ \mathbf{Glu(5)} \end{pmatrix} \left(\frac{\frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{v3}}{\mathbf{2}\boldsymbol{v2} + \frac{\mathbf{5}}{\mathbf{3}}\boldsymbol{v3}} \right) + \begin{pmatrix} \mathbf{Glu(3)} \\ \mathbf{Glu(2)} \\ \mathbf{Glu(1)} \end{pmatrix} \left(\frac{\boldsymbol{v2}}{\mathbf{2}\boldsymbol{v2} + \frac{\mathbf{5}}{\mathbf{3}}\boldsymbol{v3}} \right) + \begin{pmatrix} \mathbf{Glu(2)} \\ \mathbf{Glu(3)} \\ \mathbf{Glu(2)} \end{pmatrix} \left(\frac{\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{v3}}{\mathbf{2}\boldsymbol{v2} + \frac{\mathbf{5}}{\mathbf{3}}\boldsymbol{v3}} \right)$$